

4 Statistics and probability

Activity: The Monty Hall problem investigation (Teacher version)

A game for 2 players

One person is the host; the other person is the contestant.

- The host looks at the three cards and memorises which is the car and which are the goats.
- The host arranges the three cards in front of the contestant.
- The contestant picks a card but does not look at it. Ideally, as the contestant, you would pick the car on your first attempt.
- The host reveals one of the **goat** cards to the contestant.
- The contestant has the option to switch to the third card OR stick with their original choice.
- Record what the contestant won. (Write the item in the table below). Assume the car is a win. If they stick with the car then they have effectively won that round.

At the end of the 30 rounds, have you spotted a pattern?

	Stick or Switch	
Round Number	Stick	Switch
1		
2		
3		
4		
5		
6		
7		
8		
9		

10		
11		
12		
13		
14		
15		
16		
17		
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20		
21		
22		
23		
24		
25		
26		
27		
28		
29		
30		

Teacher Notes

Is it advantageous for the contestant to SWITCH doors, or is the probability of winning the car the same whether they STICK or SWITCH? (Assume that the host selects a card to reveal, from those available, with equal probability.)

Without loss of generality, let events A, B, C correspond to the prize being on the selected, revealed and remaining card respectively, and let H_B denote the event that the host reveals card B .

You want to compare $P(A|H_B)$ (STICK) with $P(C|H_B)$ (STICK).

$P(A) = P(B) = P(C) = \frac{1}{3}$, and you are given that $P(H_B|A) = \frac{1}{2}$, $P(H_B|B) = 0$ and $P(H_B|C) = 1$. Then the general version of Bayes' theorem gives:

$$\begin{aligned} P(A|H_B) &= \frac{P(H_B|A)P(A)}{P(H_B)} = \frac{P(H_B|A)P(A)}{P(H_B|A)P(A) + P(H_B|B)P(B) + P(H_B|C)P(C)} \\ &= \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2} \times \frac{1}{3} + 0 + \frac{1}{3}} = \frac{1}{3} \end{aligned}$$

So $P(A|H_B) = \frac{1}{3}$, and $P(C|H_B) = \frac{2}{3}$, leading to the conclusion that it is advantageous to SWITCH.